ALGEBRA REFERENCES

# Chapter P

**Absolute Value:** Denoted |*a*|, is the distance of the point *a* from the origin on the real number line and is given by



**Polynomial in One Variable:** Any expression of the form



*an*≠ 0 and where *n* is a nonnegative integer called the degree of the polynomial.

**Principal *n*th root:** If *n* is an even integer and *a* > 0, then *b* is the principal *n*th root of *a*, denoted  if and only if *b* > 0 and *bn* = *a*.

**Rational Expression:** A fraction in which the numerator and denominator are polynomials, and the denominator does not equal the zero polynomial.

**Complex Number:** Any number of the form *a* + *bi* where *a* and *b* are real numbers and .

# Chapter 1

**Quadratic Equation:** An equation that can be written in the form *ax*2 + *bx* + *c* = 0 where *a*, *b*, and *c* are real numbers and *a* ≠ 0.

**Zero Product Principle:** If *A* and *B* are algebraic expressions such that *AB* = 0, then *A* = 0 or *B* = 0.

**Quadratic Formula**: If *ax*2 + *bx* + *c* = 0, *a* ≠ 0, then





**Direct Variation:** The variable *y* varies directly as the variable *x* if and only if *y* = *kx* where *k* is a constant.

**Inverse Variation:** The variable *y* varies inversely as the variable *x* if and only if  where *k* is a constant.

# Chapter 2

**Distance Formula:** The distance *d*(*P*1, *P*2) between the points *P*1(*x*1, *y*1) and *P*2(*x*2, *y*2) is

.

**Proof**. The points *P*(*x*1, *y*1), *Q*(*x*2, *y*2), and *R*(*x*2, *y*1) form a right triangle.

We can apply the Pythagorean Theorem as follows:

*d*2 = *d*12 + *d*22

*d*2 = (*x*2 – *x*1)2 + (*y*2 – *y*1)2

Since distance is always nonnegative, 

**Function:** A set of ordered pairs in which no two ordered pairs have the same first coordinate and different second coordinates.

**Slope:** The slope m of the line passing through the points and  with  is given by



**One-to-One Function:** A function is a one-to-one function if and only if *f*(*a*) = *f*(*b*) implies

*wa* = *b*.

**Quadratic Function:** A quadratic function of *x* is a function that can be represented by and equation of the form *f*(*x*) = *ax*2 + *bx* + *c* where *a*, *b*, and *c* are real numbers and *a* ≠ 0.

# Chapter 3

**Remainder Theorem:** If a polynomial *P*(*x*) is divided by *x* – *r*, the remainder is *P*(*r*).

**Proof.** The division of a polynomial *P*(*x*) by *x* – *r* results in a quotient *Q*(*x*) and a constant remainder *R* such that

*P*(*x*) = (*x* – *r*) *Q*(*x*) + *R*

Since the identity holds for all real values of *x*, it must hold when *x* = *r*. Consequently,

*P*(*r*) = (*r* – *r*) *Q*(*r*) + *R* = 0 · *Q*(*r*) + *R* = *R*

**Factor Theorem:** A polynomial *P*(*x*) has a factor *x* – *r* if and only if *P*(*r*) = 0.

**Proof.** Part 1: Let *x* – *r* be a factor of *P*(*x*), then by the definition of a factor

 = *Q*(*x*) + *R*, where *R* = 0.

By the remainder theorem, *R* = *P*(*r*), therefore, *P*(*r*) = 0.

Part 2. Let *P*(*r*) = 0, then by the remainder theorem, *R* = 0.Therefore,

*P*(*x*) = (*x* – *r*) *Q*(*x*)

For some polynomial *Q*(*x*) of degree one less than that of *P*(*x*). By the definition of factor, *x* – *r* is then a factor of *P*(*x*).

**Intermediate Value Theorem:** If *a* < *b* and *P* is a polynomial function such that *P*(*a*) ≠ *P*(*b*), then *P* takes on every value between *P*(*a*) and *P*(*b*) in the interval [*a*, *b*].

**Rational Zero (Roots) Theorem:** If the coefficients of the polynomial

, *an* ≠ 0

are all integers and  ≠ 0 is a rational zero reduced to lowest terms, then

1. *p* is a factor of the constant term *a*0, and
2. *q* is a factor of the leading coefficient *an*.

**Fundamental Theorem of Algebra:** Every polynomial of degree *n* ≥ 1 has at least one zero among the complex numbers.

# Chapter 4

**Inverse Function:** If the ordered pairs of a function *g* are the ordered pairs of a function *f* with the order of the coordinates reversed, then *g* is the inverse function of *f*.

**Exponential Function:** The exponential function with base *b* is defined by

*f*(*x*) = *bx*

where *b* > 0, *b* ≠ 1, and *x* is a real number.

**Logarithmic Function:** If *x* > 0 and *b* is a positive constant (*b* ≠ 1), then

** if and only if .

**Properties of Logarithms**

1. *Product property* 
2. *Quotient property* 
3. *Power property* 

**Change of Base Formula:** If x, a, and b are positive real numbers with a ≠ 1 and b ≠ 1, then

