Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *Let f be a continuous function, then the derivative of f, denoted $f'(x)$, is given by*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Theorem 1. *Let f, g be differentiable functions for all $x \in X$, then $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$ for all $x \in X$.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Using induction, prove the following.

Theorem 2. *For any* $n \in \mathbb{Z}^{>}$, $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$.

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. $n! = 1 \cdot 2 \cdot \ldots (n-1) \cdot n$ *for any* $n \in \mathbb{Z}^{>}$ *and* $0! = 1$

Definition. *Let* $n, r \in \mathbb{Z}^{\geq}$ *, then* $\displaystyle\binom{n}{r} = \begin{cases} \dfrac{n!}{r!(n-r)!} & \text{for } 0 \leq r \leq n \\ 0 & \text{else} \end{cases}$

Theorem 3. $\displaystyle\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ *for any* $n \in \mathbb{Z}^{+}$ *and* $r = 1, 2, \ldots, n-1$.

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *The absolute value of a real number x, denoted $|x|$, is given by* $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$.

Use proof by cases (consider all cases for a, b that generate different possible cases for $|a+b|$). Other proofs types exist for this problem but will not receive full credit.

Theorem 4. *Let $a, b \in \mathbb{R}$, then $|a+b| \leq |a| + |b|$*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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(You must prove both parts of the theorem for full credit).

Theorem 5. *Prove the finite sum for the geometric series:*

$$\sum_{k=0}^{n} ar^k = \frac{a(1-r^{n+1})}{1-r}$$

Using the above result for the finite sum, extend it to the infinite sum.

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \text{ if } |r| \in (0,1)$$

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *A function, $f(x)$, can be a density function of a random variable (RV) X iff it satisfies*

$$f(x) \geq 0 \text{ for all } x, \qquad \text{and} \qquad \sum_{\text{all } x} f(x) = 1 \text{ (if } X \text{ is discrete)} \left(\text{or} \int_{\text{all } x} f(x)dx = 1 \text{ (if } X \text{ is continuous)} \right).$$

Definition. *If X is a RV with density function $f(x)$, then the expected value (aka mean) of a function of X, $g(X)$, is given by*

$$E[g(X)] = \sum_x g(x) \cdot f(x) \text{ (if } X \text{ is discrete)} \left(\text{or } E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)dx \text{ (if } X \text{ is continuous)} \right)$$

Definition. *Maclaurin Expansion of $f(x)$: $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}(x-0)^k$*

Write out the Maclaurin expansion of $h(\lambda) = e^{\lambda}$.

$h(\lambda) =$

Theorem 6. *If $X \sim Poisson(\lambda)$, then $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for some $\lambda > 0$ and for any $x = 0, 1, \ldots$.*

Show that $E[X] = \lambda$.

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *Let $u, v \in \mathbb{R}^3$ such that $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$, then*

$$u \times v = (u_2v_3 - u_3v_2, -u_1v_3 + u_3v_1, u_1v_2 - u_2v_1).$$

Theorem. *Given $u, v \in \mathbb{R}^3$ and let θ be the angle between u and v, then $||u \times v|| = ||u||\,||v|| \sin\theta$.*

Given the above, prove the following:

Theorem 7. *Let $u, v \in \mathbb{R}^3$ and $c \in \mathbb{R}$, then $u \times v = 0$ (this is called the cross product) if and only if u and v are scalar multiples of each other.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Theorem. *Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.*

Given the Extreme Value theorem, prove Rolle's theorem (do not use the MVT in your proof).

Theorem 8. *Rolle's Theorem: Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$ then there exists $c \in (a, b)$ such that $f'(c) = 0$.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *An $m \times n$ matrix, A, is a rectangular array of (real) numbers composed of m rows and n columns (aka $A \in \mathbb{R}^{m\times n}$) where $[A]_{ij} = a_{ij}$ is the entry in i^{th} row and j^{th} column.*

Definition. *If $A, B \in \mathbb{R}^{m\times n}$, then $A + B \in \mathbb{R}^{m\times n}$, where $[A + B]_{ij} = a_{ij} + b_{ij}$ for all i, j.*

Theorem 9. *Show that $A \in \mathbb{R}^{m\times n}$ always has an additive inverse for any m, n.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *Division Algorithm for Polynomials: Let $p(x)$ be a polynomial of degree n and $d(x)$ be a polynomial of degree m with $1 \leq m < n$. Then there exists unique polynomials $q(x)$ and $r(x)$ with the degree of $r(x)$ being less than m where*

$$p(x) = d(x) \cdot q(x) + r(x).$$

The polynomial $p(x)$ is called the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.

Theorem 10. *Remainder Theorem: Let $c \in \mathbb{R}$ be a fixed value and let $P(x)$ be a polynomial. Then if $P(x)$ is divided by $x - c$, the remainder is $P(c)$.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *The Jacobian, J, is the determinant of the matrix of mixed partials. Let $x = g(u, v, w), y = h(u, v, w)$, and $z = k(u, v, w)$, then*

$$J = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} \qquad \textit{where} \qquad a_b = \frac{\partial}{\partial b} a.$$

Definition. *The determinant of a 2×2 matrix is given by* $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

Definition. *The determinant of a 3×3 matrix is given by*

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a \begin{vmatrix} e & f \\ h & k \end{vmatrix} - b \begin{vmatrix} d & f \\ g & k \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}.$$

Theorem. *Under the proper givens, the following is the change of variables for triple integrals:*

$$\int\int\int f(x, y, z) dxdydz = \int\int\int f(g(u, v, w), h(u, v, w), k(u, v, w)) \, |J| \, dudvdw.$$

Given the above, prove the following.

Theorem 11. *Assume the proper givens. If $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta$, and $z = \rho \cos \phi$, then*

$$dxdydz = \rho^2 \sin \phi \; d\rho d\phi d\theta.$$

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *The Fibonacci numbers are defined by* $f_1 = f_2 = 1$ *and* $f_{n+2} = f_{n+1} + f_n$ *for* $n = 1, 2, \ldots$.

Theorem 12. *Prove that the* n^{th} *Fibonacci number* f_n *satisfies* $f_n < 2^n$.

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Theorem 13. *Prove that the composition of any two decreasing functions is increasing.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *A relation $\mathcal{R}$ defined on a set A is called an equivalence relation if it has the following three properties, where a, b, and c denote arbitrary elements of A:*

1. $a\mathcal{R}a$
2. *If* $a\mathcal{R}b$ *then* $b\mathcal{R}a$.
3. *If* $a\mathcal{R}b$ *and* $b\mathcal{R}c$, *then* $a\mathcal{R}c$.

Prove the following theorem:

Theorem 14. *For any $a, b \in \mathbb{R}$, if $a - b \in \mathbb{Z}$ then $a\mathcal{R}b$ is an equivalence relation.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Theorem 15. *If u and v are functions of x and have continuous derivatives then*

$$\int u\, dv = uv - \int v\, du.$$

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *When $a = qn + r$ where q is the quotient and r is the remainder upon dividing a by n, we write*

$$a \mod n = r.$$

Definition. *If $n \neq 0$, $a \in \mathbb{Z}$, then n divides a, denoted $n|a$, if $\exists m \in \mathbb{Z}$ such that $a = nm$.*

Theorem. *If $a, b \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$, then $a \mod n = b \mod n$ iff $n|a - b$.*

Given the above, prove the following.

Theorem 16. *For any $n \in \mathbb{Z}$, $n^3 \mod 6 = n \mod 6$.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *A vector space V is a set that is closed under finite vector addition and scalar multiplication. The basic example is n-dimensional Euclidean space $\mathbb{R}^n$, where every element is represented by a list of n real numbers, scalars are real numbers, addition is component-wise, and scalar multiplication is multiplication on each term separately.*

Definition. *If T is a mapping (transformation) from vector space, V, to vector space W, then T is a linear transformation iff $T[\mathbf{v}_1 + \mathbf{v}_2] = T[\mathbf{v}_1] + T[\mathbf{v}_2]$ and $T[c\mathbf{v}] = cT[\mathbf{v}]$ for $c \in \mathbb{R}$*

Theorem 17. *Prove that $T\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right] = 2a + (b+c)x - 2(d-c)x^2$ is a linear transformation.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *A square matrix I is the identity matrix if $i_{ij} = 1$ for $i = j$ and 0 otherwise.*

Definition. *If $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times p}$, then AB is an $m \times p$ matrix where $[AB]_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$ for all i, j (dot product rows of A with columns of B).*

Definition. *A matrix $B \in \mathbb{R}^{n\times n}$ is an inverse for $A \in \mathbb{R}^{n\times n}$, denoted $B = A^{-1}$ iff $AB = BA = I$.*

Theorem 18. *If $A \in \mathbb{R}^{n\times n}$ has a multiplicative inverse, then it is unique.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Prove the following:

Theorem 19. *For every odd integer x, there exists $y \in \mathbb{Z}$ such that $x^2 = 8y + 1$.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Theorem 20. *Prove the following statement: If $x, y \in \mathbb{Z}$, and $x^2 + y^2$ is even, then $x + y$ is even.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Definition. *A pair of objects (X, d) consisting of a non-empty set X and a function $d : X \times X \to R$, where R is the set of real numbers, is called a metric space provided that:*

1. $d(x, y) \geq 0, x, y \in X$;
2. $d(x, y) = 0$ *iff* $x = y$, $x, y \in X$;
3. $d(x, y) = d(y, x), x, y \in X$;
4. $d(x, z) \leq d(x, y) + d(y, z), x, y, z \in X$.

The function d is called a distance function or metric on X, and the set X is called the underlying set.

Theorem 21. *Let X be a set. For $x, y \in X$ define the function d by $d(x, x) = 0$ and $d(x, y) = 1$ for $x \neq y$. Prove that (X, d) is a metric space.*

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page, use white space and indentations to add clarity to your proof). Clearly express each step of the logic of the proof (this may require commentary in English).

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Using induction, prove the following.

Theorem 22. *For any* $n \in \mathbb{Z}^{>}$, $\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$. *Recall* $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$.