Your proof will be graded for presentation/clarity (up to 10% deduction for poor presentation of a correct proof) as well as accuracy/completeness. Failure to submit at least one proof per week will result in a 2 pt deduction (approximately 0.5%) of your overall course grade.

Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. Let f be a continuous function, then the derivative of f, denoted f'(x), is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Theorem 1. Let f, g be differentiable functions for all $x \in X$, then $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$ for all $x \in X$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

Th F Sa Su M T W Th (due)

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. $n! = 1 \cdot 2 \cdot \ldots (n-1) \cdot n$ for any $n \in \mathbb{Z}$

Definition. Let $n, r \in \mathbb{Z}$, then $\binom{n}{r} = \begin{cases} \frac{n!}{r!(n-r)!} & \text{for } 0 \leq r \leq n \\ 0 & \text{else} \end{cases}$

Theorem 3. $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ for any $n \in \mathbb{Z}^+$ and $r = 1, 2, \dots, n-1$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. The absolute value of a real number x, denoted |x|, is given by $|x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$.

Use proof by cases (consider all cases for a, b that generate different possible cases for |a + b|). Other proofs types exist for this problem but will not receive full credit.

Theorem 4. Let $a, b \in \mathbb{R}$, then $|a + b| \leq |a| + |b|$

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 5. Prove the finite sum for the geometric series:

$$\sum_{k=0}^{n} ar^{k} = \frac{a(1-r^{n+1})}{1-r}$$

(You must prove both theorems for full credit).

Theorem. Using the result from above for the finite sum, and extend it to the infinite sum.

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \ if \, |r| \in (0,1)$$

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. A function, f(x), can be a density function of a random variable (RV) X iff it satisfies

$$f(x) \ge 0 \text{ for all } x, \quad \text{and} \quad \sum_{all \ x} f(x) = 1 \text{ (if } X \text{ is discrete)} \left(\begin{array}{c} or \int f(x) dx = 1 \text{ (if } X \text{ is continuous)} \\ all \ x \end{array} \right)$$

Definition. If X is a RV with density function f(x), then the expected value (aka mean) of X is given by

$$E[X] = \sum_{x} x \cdot f(x) \text{ (if } X \text{ is discrete)} \left(\text{ or } E[X] = \int_{-\infty}^{\infty} x \cdot f(x) dx \text{ (if } X \text{ is continuous)} \right)$$

Definition. Maclaurin Expansion: $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} (x-0)^k$

Theorem 6. If $X \sim Poisson(\lambda)$, then $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for some $\lambda > 0$ and for any $x = 0, 1, \dots$ Show that $E[X] = \lambda$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. Let $u, v \in \mathbb{R}^3$ such that $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$, then

 $u \times v = (u_2v_3 - u_3v_2, -u_1v_3 + u_3v_1, u_1v_2 - u_2v_1).$

Theorem. Given $u, v \in \mathbb{R}^3$ and let θ be the angle between u and v, then $||u \times v|| = ||u|| ||v|| \sin \theta$.

Given the above, prove the following:

Theorem 7. Let $u, v \in \mathbb{R}^3$ and $c \in \mathbb{R}$, then $u \times v = 0$ (this is called the cross product) if and only if u and v are scalar multiples of each other.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem. Extreme Value Theorem: If f is continuous on a closed interval [a,b], then f has both a minimum and a maximum on the interval.

Given the Extreme Value theorem, prove Rolle's theorem.

Theorem 8. Rolle's Theorem Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a) = f(b) then there exists $c \in (a, b)$ such that f'(c) = 0.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. An $m \times n$ matrix, A, is a rectangular array of (real) numbers composed of m rows and n columns (aka $A \in \mathbb{R}^{m \times n}$) where $[A]_{ij} = a_{ij}$ is the entry in i^{th} row and j^{th} column.

Definition. If $A, B \in \mathbb{R}^{m \times n}$, then $A + B \in \mathbb{R}^{m \times n}$, where $[A + B]_{ij} = a_{ij} + b_{ij}$ for all i, j.

Theorem 9. Show that $A \in \mathbb{R}^{m \times n}$ always has an additive inverse for any m, n.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 10. Let $u, v, w \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$. Show that c(u+v) = cu + cv and ||cv|| = |c| ||v||.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. Division Algorithm for Polynomials: Let p(x) be a polynomial of degree n and d(x) be a polynomial of degree m with $1 \le m < n$. Then there exists unique polynomials q(x) and r(x) with the degree of r(x) being less than m where

$$p(x) = d(x) \cdot q(x) + r(x).$$

The polynomial p(x) is called the dividend, d(x) is the divisor, q(x) is the quotient, and r(x) is the remainder.

Theorem 11. Remainder Theorem: If a polynomial P(x) is divided by x - r, the remainder is P(r).

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. The Fibonacci numbers are defined by $f_1 = f_2 = 1$ ad $f_{n+2} = f_{n+1} + f_n$ for n = 1, 2, ...**Theorem 12.** Prove that the n^{th} Fibonacci number f_n satisfies $f_n < 2^n$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 13. Prove that the composition of any two decreasing functions is increasing.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 14. If u and v are functions of x and have continuous derivatives then

$$\int u \, dv = uv - \int v \, du.$$

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. Let $s, t \in \mathbb{Z}$, then $t \neq 0$ is a divisor of s iff $\exists u \in \mathbb{Z}$ such that s = tu and we write t|s.

Theorem. For any nonzero $a, b \in \mathbb{Z}$, there exists $s, t \in \mathbb{Z}$ such that the greatest common divisor of a and b, gcd(a, b) = as + bt.

Theorem 15. Euclid's Lemma: If p is a prime such that p|ab, then p|a or p|b.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Definition. When a = qn + r where q is the quotient and r is the remainder upon dividing a by n, we write a mod n = r or $a = r \mod n$.

Theorem. If $a, b \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$, then $a \mod n = b \mod n$ iff n|a - b.

Given the above, prove the following.

Theorem 16. For any $n \in \mathbb{Z}$, $n^3 \mod 6 = n \mod 6$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. A vector space V is a set that is closed under finite vector addition and scalar multiplication. The basic example is n-dimensional Euclidean space \mathbb{R}^n , where every element is represented by a list of n real numbers, scalars are real numbers, addition is component-wise, and scalar multiplication is multiplication on each term separately.

Definition. If T is a mapping (transformation) from vector space, V, to vector space W, then T is a linear transformation iff $T[\mathbf{v}_1 + \mathbf{v}_2] = T[\mathbf{v}_1] + T[\mathbf{v}_2]$ and $T[c\mathbf{v}] = cT[\mathbf{v}]$ for $c \in \mathbb{R}$

Theorem 17. Prove that $T\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right] = 2a + (b+c)x - 2(d-c)x^2$ is a linear transformation.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. A square matrix I is the identify matrix if $i_{ij} = 1$ for i = j and 0 otherwise.

Definition. If $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$, then AB is an $m \times p$ matrix where $[AB]_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$ for all i, j (dot product rows of A with columns of B).

Definition. A matrix $B \in \mathbb{R}^{n \times n}$ is an inverse for $A \in \mathbb{R}^{n \times n}$, denoted $B = A^{-1}$ iff AB = BA = I.

Theorem 18. If $A \in \mathbb{R}^{n \times n}$ has a multiplicative inverse, then it is unique.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Definition. The Jacobian, J, is the determinant of the matrix of mixed partials. Let x = g(u, v, w), y = h(u, v, w), and z = k(u, v, w), then

$$J = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} \qquad where \qquad a_b = \frac{\partial}{\partial b}a$$

Definition. The determinant of a 2×2 matrix is given by $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Definition. The determinant of a 3×3 matrix is given by

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a \begin{vmatrix} e & f \\ h & k \end{vmatrix} - b \begin{vmatrix} d & f \\ g & k \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}.$$

Theorem. Under the proper givens, the following is the change of variables for triple integrals:

$$\int \int \int f(x,y,z) dx dy dz = \int \int \int \int f(g(u,v,w), h(u,v,w), k(u,v,w)) |J| du dv dw$$

Given the above, prove the following.

Theorem 19. Assume the proper givens. If $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, and $z = \rho \cos \phi$, then

$$dxdydz = \rho^2 \sin \phi \, d\rho d\phi d\theta.$$

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Prove the following:

4

Theorem 20. For every odd integer x, there exists $y \in \mathbb{Z}$ such that $x^2 = 8y + 1$.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

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Neatly present a proof for the numbered theorem below (transcribe your final proof to this page (front and/or back as needed), use white space and indentations to add clarity to your proof). Be sure to clearly express each step of the logic of the proof.

Theorem 21. Prove the following statement: If $x, y \in \mathbb{Z}$, and $x^2 + y^2$ is even, then x + y is even.

Developing a proof takes time and pondering, so starting early is expected (no help from your professor will be given on the day the assignment is due). To hold you accountable for this expectation, indicate the day of the week you started working on this proof:

Th F Sa Su M T W Th (due)

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