# *Ma 300 Theory*

There will be several uniqueness proofs during the semester. Know these and in general know how to do a uniqueness type of proof.

Chapter 1:

1. Inconsistent and consistent.
2. Elementary Row operations.
3. Row Echelon Form.
4. Reduced Row Echelon form.

Chapter 2:

1. Matrix addition, multiplication, and scalar multiplication.
2. Identity Matrix and Inverse of a matrix.
3. Elementary Matrix
4. Row Equivalent Matrices

Proofs:

1. Properties of Matrix Addition and Scalar Multiplication(Thm 2.1).
2. Properties of Matrix Multiplication(Thm 2.3) Specifically the 2 Distributive properties.
3. **Uniqueness of the Inverse Matrix. (Thm 2.7)**
4. The inverse of a product (Thm 2.9)
5. Cancellation properties(Thm 2.10)
6. Thm 2.14

Chapter 3

1. Minor and Cofactor
2. Theorem 3.3
3. Equivalent Conditions for a Nonsingular Matrix.
4. Eigenvalue and Eigenvector
5. Characteristic Equation

Proofs

1. Theorem 3.3
2. Theorem 3.5: Determinant of a Product

Chapter 4: ( The Big One)

1. Definition of Vector Space
2. Definition of Subspace
3. Linear combination(of a set of vectors)
4. Span of a set

5. Spanning set
6. Spans
7. Linear independence
8. Basis(of a subspace)
9. Dimension of a Vector Space
10. Row Space of a Matrix
11. Column Space of a Matrix
12. Rank of a Matrix
13. Null Space of a Matrix.
14. Theorem 4.17 (Key Theorem)
15. Coordinate vector or x relative to (basis) B
16. Transition Matrix

Proofs

1. Be able to show a set under operations defined is or is not a vector space.
2. Properties of scalar multiplication(Theorem 4.4)
3. Test for a subspace (Theorem 4.5)
4. Be able to show a subset of a vector space is or is not a subspace.
5. Key subspaces(Span of a set, Row space, column space, null space. Be sure you can show each is a subspace)
6. Theorem 4.8
7. **Uniqueness of Basis Representation: Theorem 4.9**
8. Theorem 4.10
9. Theorem 4.11

Chapter 5: We will focus on primarily 2 Inner product spaces ($R^n$ and C[a, b]) Know both of these

1. Dot Product
2. Definition of Inner Product:
3. Definitions of norm(length) distance, angle, and orthononality in a generalized Inner product space.
4. Cauchy Schwarz Inequality
5. Orthonormal
6. Gram Schmidt Orthonormalization Process
7. Fourier Coefficients

Proofs:

1. **Cauchy Schwarz Inequality(in a generalize Inner product space)**
2. Triangular Inequality and Pythagorean Theorem.
3. Coordinates Relatives to an Orthonomal Basis(Them 5.11)

Key Application: Finding the Fourier coefficients and a Fourier Approximation in C{0, $2\pi$]

Chapter 6

1. Linear Transformation

2. Domain, Range and Kernal
3. One to one, and Onto
4. Isomorphism
5. Similar Matrices

Proofs(and Skills)

1. Properties of Linear Transformations(Theorem 6.1)
2. Know how to prove a transformation is linear
3. Know for to prove a function(and linear transformation) is 1 -1
4. Know how to prove a function(and linear transformation) is onto
5. Prove the Kernal is a subspace of the Domain.
6. Given a Transformation for $R^n$ to $R^m$, be able to write it as a matrix operation.
7. Find Transition matrices with respect to different basis.