BOB JONES UNIVERSITY
Department of Mathematics

Ma 311 ~ Linear Algebra

Spring 2025

Instructor: Dr. Laurel Carpenter
Office: AL 46
Office Hours: schedule via https://calendly.com/llcarpen
Email: llcarpen@bju.edu
Preferred Contact: MS Teams: personal message via personal chat,
course content questions via this course's Q&A channel

Lecture: MWF 2:00-2:50, AL 301

Textbook: Larson, *Elementary Linear Algebra,* 8th edition, Cengage, 2017. ISBN 9781305658004
Calculator Requirements: A TI-89 or TI-Nspire is required (TI-Nspire CAS is recommended)

Catalog Description: Vector, vector functions, linear functions, solutions of systems of linear equations, matrices, determinants, and eigenvalues.

Course Context: Linear Algebra is required for programs in Mathematics, Physics, Computer Science, Actuarial Science, and Mathematics Education and is recommended for Engineering.

This course supports the following Core Goals (IG) of this institution, goals (MS) of the Division of Mathematical Sciences, and program learning outcomes (MM) for the mathematics major in which, upon degree completion, the student will…

IG 2: Communicate effectively by various means in a variety of contexts.
IG 3: Understand the human experience within the framework of … natural sciences, and mathematics.
IG 4: Analyze, evaluate, and synthesize information and ideas.
IG 5: Solve problems through critical and creative thinking, working independently or collaboratively.
MS 1: Understand the essential theory of mathematics… and appropriately apply the theory in solving problems.
MS 2: Use critical-thinking/analytical skills to understand mathematical… problems and design solutions with the aid of appropriate tools.
MS 3: Apply an understanding of how mathematics… can be used as a tool to examine the natural universe.
MM 1: Progress logically from premises to valid conclusions in a variety of mathematical contexts.
MM 2: Apply mathematics to model real-life situations.
MM 3: Select and use technology for understanding, as well as a labor-saving or problem-solving tool.
MM 4: Construct a biblically consistent philosophy of mathematics.

Course Learning Outcomes: This course is designed to…

1. Develop competency at applying linear systems and matrix solution techniques in a variety of contexts including engineering, computing, and the natural sciences.
2. Provide a foundation in vector and matrix theory that supports related areas such as analysis and differential equations, operations research, modern geometry, stochastics, cryptography, and graph theory.
3. Introduce the field of abstract algebra including important concepts in vector spaces and linear transformations.
4. Sharpen critical-, analytical-, and creative-thinking while applying abstract ideas to scenarios in the natural world.
5. Develop concise use of language and logical thought in proving algebraic properties and theorems.
6. Foster instructor-independent learning of new mathematical concepts, approaches, and applications.
7. Encourage the communication of mathematical concepts clearly to various audiences both orally as well as in writing.

Objectives:

	The student will be able to…	Course Goals Supported	Course Content	Assessment
1.	Reduce a matrix to an REF or the RREF either by hand or using technology	CG 1, CG 3, CG 5	Chapter 1	HW, Test
2.	Use matrices (by conversion to RREF) to solve linear systems	CG 3, CG 4	Chapter 1	HW, Test
3.	Apply linear systems to problems such as curve fitting, network analysis, and chemical reactions	CG 2, CG 4, CG 5	Chapter 1	HW, Project
4.	Perform matrix operations including matrix addition, matrix multiplication, scalar-matrix multiplication, and transposition	CG 1, CG 3, CG 5	Chapter 2	HW, Quiz, Test
5.	Identify the algebraic properties of matrices	CG 1, CG 3, CG 5, CG 7	Chapter 2	HW, Quiz, Test,
6.	Factor matrices (when applicable) into elementary matrices and/or into LU form	CG 1, CG 2, CG 3, CG 4, CG 5	Chapters 2 and 3	HW, Test
7.	Apply matrices to problems in stochastics, cryptography, and Leontief input-output models	CG 2, CG 4, CG 5	Chapter 2	HW, Project
8.	Construct cofactors and calculate determinants of matrices both by hand and using technology	CG 1, CG 3, CG 5	Chapter 3	HW, Quiz, Test
9.	Construct inverse matrices both by hand (using elementary matrices, using adjoints) and using technology		Chapters 2 and 3	HW, Quiz, Test
10.	Identify when a square matrix is invertible (using the Equivalent Conditions for a Nonsingular Matrix)	CG 1, CG 2, CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 3	HW, Test
11.	Apply determinants to problems of area and volume in real n-space as well as to applications using Cramer's Rule		Chapter 3	HW, Project
12.	Determine if a given structure is a vector space, a subspace of a vector space, and/or an inner-product space	CG 1, CG 3, CG 5	Chapters 4 and 5	HW, Quiz, Test
13.	Determine if a given set of vectors is linearly independent, a spanning set, and/or a basis	CG1, CG 3, CG 5, CG 7	Chapter 4	HW, Quiz, Test
14.	Define bases (and calculate dimensions) for the row-space (rank), column-space, and null-space (nullity) of a matrix and articulate the principle of duality	CG1, CG 3, CG 5, CG 7	Chapter 4	HW, Quiz, Test
15.	Determine coordinate and transition matrices in real n-space	CG 1, CG 2 CG 3, CG 4, CG 5, CG 7	Chapter 4	HW, Test
16.	Calculate (in an inner-product space) the norm of a vector, the distance between vectors, and the angle between vectors	CG 1, CG 2 CG 3, CG 4 CG 5, CG 6, CG 7	Chapter 5	HW, Test
17.	Construct orthogonal projections and orthonormal basis for a given basis in an inner product space using the Gram- Schmidt process	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 5	HW, Test
18.	Construct a Fourier approximation to a polynomial	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 5	HW, Test
19.	Identify when a relation is a linear transformation and if the linear transformation is 1-1 and/or onto.	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 6	HW, Quiz, Test
20.	Determine bases for the kernal, domain, and range of a linear transformation	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 6	HW, Test
21.	Represent a linear transformation as a matrix.	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 6	HW, Test
22.	Determine if two matrices represent the same linear transformation	CG 1, CG 2 CG 3, CG 4, CG 5, CG 6, CG 7	Chapter 6	HW, Test
23.	Determine (and interpret) eigenvectors and associated eigenvalues of linear transformations.	CG 2, CG4	Chapter 7	HW, Test

Grading and Assessment

Grading Scale:

Final grades will be assigned according to a standard 10 percentage-point scale calculated out of the total points available during the semester. Percentages will be rounded to the nearest whole percentage when determining final grades: A (90% - 100%), B (80% - 89%), C(70% - 79%), D(60% - 69%).

Grades are determined by total points made up of the following categories:

	Point Value	**Note**
Exercises and Proofs	200	10 exercise checks @ 10 points each 4 proof portfolio reviews @ 25 points each
Quizzes	200	10 quizzes @ 20 points each
Tests	340	4 tests @ 85 points each
Final Exam	160	1 cumulative exam @ 160 points
Projects	100	3 project reports @ 25 points each 1 project presentation @ 25 points
Total Point Value	**1000**	

Study, Exercises, and Proofs:

- Students are expected to make wise choices in the ways they engage with their study for this course. Study should include (but may not be limited to) engagement with exercises from the textbook, practice proving theory, and memorization.
- Suggested Exercises are included on the course schedule. Students are expected to engage with as many exercises as needed not only to gain understanding of the concepts and skills but also to build automaticity.
- **Exercise Checks:** Students will be expected to complete* specified exercises and bring them to class as part of their Exercises and Proofs assessment. Exercise checks may occur at random, so students should stay up to date on their exercises. (*Completion includes checking answers, correcting incorrect work, and presenting work in finished form.)
- **Proof Portfolios:** Students will be expected to compile a portfolio of all theory (definitions, properties, theorems, etc.) as well as proofs for all properties and theorems in this course. Portfolios will be due in the instructor's credenza by the end of day on the day of the unit test and on the last class day of the semester.

Quizzes, Tests, and Final Exam:

- Quizzes, tests, and final exam are listed on the course syllabus.
- Quizzes will be in class and closed book. Some quizzes will be without calculator.
- Tests will be in class, closed book, and will allow the use of a TI-89 or TI-NSpire calculator.

Project Reports and Presentations:

- Report due dates are listed in the course syllabus. Presentation dates will be assigned near the end of the semester.
- Each student will be expected to complete three project reports, one theory project and two application projects. Reports are to be submitted by individuals not by teams.
- Students will also be expected to work in teams to present the contents of one of their application project topics.
- A student not participating in their own team's presentation will receive a 0 on the presentation. Any student missing another team's presentation will incur a 10% penalty on their own presentation grade for each missed presentation.

Late or Missing Assessments:

- **Permission:** A student wishing to receive credit for a late assignment must seek the permission of their instructor.
- **Late Penalty:** Unless otherwise stated below, late assessments in this course will be penalized $(n \cdot 10)$ percentage points per day if taken within three calendar days starting at the time the assignment is due where n is the number of times a late penalty has been granted. After which, the grade will be zero.
- Missing work will receive a grade of zero.
- Exercise Checks will not be accepted late. Any student may request that one exercise check during the semester be excused without penalty for any reason.
- Proof Portfolios may be turned in after the due date only with the instructor's permission and will incur the late penalty.
- Quizzes and Tests missed due to absence may be made up only with the instructor's permission and will incur the late penalty. In extenuating circumstances, the late penalty may be waived.
- The Final Exam, may be made up only with the Registrar's permission.
- Project Reports are subject to the late penalty if turned in late.
- Project Presentations may be delayed for extenuating circumstances only with the instructor's approval and only if time in the course allows. Delayed presentations will incur a 20% penalty.

Classroom Deportment

All meetings are to be conducted in a professional manner. That means, while in attendance students are expected to focus on course-related material and to contribute positively to the meeting. The instructor reserves the right to ask a student to leave a meeting should their attention be elsewhere (sleeping, surfing the internet, working on assignments for another class, etc.).
Professionalism includes the attitudes being conveyed. Respect is to be shown towards all in attendance. Discourse should be gracious. Critique and inquiry is to be collegial – given and received with humility, fairness, and an open-mind.

Absences:

Missed meetings will be counted as course absences. Arriving late or leaving early from a meeting will count as a partial absence. The University's attendance policy is in effect.
(See https://home.bju.edu/bju-policies/ for details.)

Academic Integrity and Artificial Intelligence Policies:

The University's Academic Integrity Policy is in effect. (See https://home.bju.edu/bju-policies/ for more details).

- **Artificial Intelligence:** Because the goal of the assignments in this course is to develop skills rather than simply to complete tasks, and because the use of AI to complete or jump-start tasks defeats that goal, students may not use generative AI tools (i.e. Chat GPT, Bing Chat, Google Bard, etc.) in this course for any assignment without the instructor's explicit permission. Should an AI tool be used with permission, its use must be documented (including the tool used, a summary of the prompts provided and the portions of the assignment that were based on AI generated work).
- **Cheating and Plagiarism:** Cheating is defined as any use of unauthorized helps. Plagiarism is defined as taking someone else's words and/or ideas and claiming them as one's own. All work done for this course must be independent and original. If information is taken from other sources (which is at times appropriate), it must be adequately cited so credit is given to whom it is due. Use standard referencing techniques as taught in En 102.
- **Originality:** Students are permitted (and encouraged) to discuss the ideas of their research but are not permitted to collaborate with anyone other than their instructor on graded assignments unless working on a collaborative effort under the explicit direction of the instructor. In which case, the instructor will

determine which assignments may be worked on and submitted jointly. Graded assignments should represent the student's own ideas and their own work and should be the product of their own thinking and efforts. A student may not use AI to generate any portion of their papers or presentations without explicit permission from their instructor (and if permission is granted it must be documented as described above).

- **Ask Your Instructor:** If you have a question about any source you are considering using, please gain your instructor's approval before using it. You are always permitted to ask your instructor for help. Any help they choose to provide is acceptable.

University Policies: We will follow University guidelines.

Copyright Policy:

Ma 311 Linear Algebra ❖ Course Schedule and Assignments ❖ Spring 2025

This assignment sheet is a tentative schedule. We may get ahead or behind; adjust as necessary.

Date	Discussion	Assessment	Suggested Exercises
Unit 1 Linear Systems, Matrices, and Determinants			
			• Exercises in parenthesis () are for the purpose of reviewing what you should already know from prior math experience. Do only what you need to refresh your knowledge. • Exercises not in parenthesis represent potentially new material. Pay attention to these. • Exercises in ***bold italic*** introduce ideas or either build or reinforce theory. Neglect these at your own peril. • Exercises that are underlined are to be turned in.
Wk 1 ---- 1/15 1/17	§ 1.1-1.3	**Quiz 1 (Elimination hand)**	1.1 #(1-66), 69, 70, 71, 72, 77, 78, 79, 81, 83, 85, ***91, 92,*** 93 1.2 #3, 7, 11, 13, 15, 17, 19-21, 22-24, 33, 35, 37, 39, 41, 43, ***50, 52, 59, 60, 63, 68*** 1.3 curve fitting: #3, 7, 11, 13, 14, 15, 37 network analysis: #21, 23, 25 partial fraction decomposition: #31, 33 polynomial zeros #***39, 40***
Wk 2 1/20 1/22 1/24	§ 2.1-2.6 **MLK Day**	no class Monday **Quiz 2 (Matrix Ops hand)**	2.1 #7, 11, 15, 17, 21, 23, 29, 31, 33, 35, 41, 43, 47, 49, 51, ***55, 57, 59, 61, 63, 65, 67, 68, 69, 70, 73, 85, 86*** 2.2 #7, 13, 15, 27, 29, 37, 38, ***41, 45, 49, 51, 53, 55, 56, 59, 69, 70, 71, 73*** 2.3 #5, 7, 13, 15, 25, 27, 41, 45, 47, ***53, 55, 57, 59, 69, 71, 72, 74, 75*** 2.4 #1-11 odd, 15, 17, 19, 21, ***23***, 25, 31, 33, 35, ***41, 42,*** 53 2.5 (TBD) stochastic matrices, Markov chains, steady states 2.6 cryptography #1, 3, 5, 7, 9
Wk 3 1/27 1/29 1/31 Wk 4 2/3 2/5 2/7	(§ 2.1-2.6 cont) § 3.1-3.4 (start § 4.1-4.8)	**Quiz 3 (Determinants hand)** **Test 1 & Portfolio**	3.1 #1, 5, 9, 15, 17, 19, 27, 29, 31, 39, 41, ***43, 44,*** 45, 47, 51, 67, 68 3.2 #1-15 odd, 21, 23, 25, 27, 31, 35, ***37, 38***, 39, 41, 45 3.3 #3, 5, 13, 17, 21, 23, 25, 33, ***37, 39, 41,*** 47, 57, ***59, 60,*** 65, ***71, 72, 75, 78*** 3.4 adjoint #1, 5, 7 Cramer's Rule #9, 13, 17, 21, ***27***
Unit 2 Vector Spaces			
Wk 5 2/10 2/12 2/14 Wk 6 2/17 2/19 2/21 Wk 7 2/24 2/26 2/28	§ 4.1-4.8 **Bible Conference** 	**Quiz 4 (Vector Spaces)** **Quiz 5 (Independence)** no class Wednesday no class Friday **Project 1 Report** **Test 2 & Portfolio**	4.1 #1, 5, 9, 13, 15, 17, 21, 23, 27, 31, 33, 37, 39, ***57, 58*** 4.2 #1-12 all, 13, 17, 21, 25, 29, 31, 33, 35, 37, 41, ***49, 50*** 4.3 #3, 4, 7, 9, 11, 17, 23, 27, 29, 33, 37, 39, ***43, 44, 54, 55*** 4.4 #1, 3, 5, 9, 13, 17, 19, 23, 25, 31, 33, 35, 37, 41, 49, 53, 55, ***57, 63, 64, 74, 76*** 4.5 #1, 3, 5, 7, 13, 15, 17, 21, 25, 27, 31, 33, 35, 37, 38, 39, 43, 45, 49, 53, 59, 61, 63, 65, ***79, 80, 81, 83*** 4.6 #5, 7, 9, 15, 17, 21, 23, 25, 33, 39, 41, 43, 47, 51, 53, 57, 59, ***67, 71, 73, 74, 75, 76*** 4.7 #7, 9, 13, 15, 17, 19, 21, 23, 27, 31, 33, 39, 41, 45, 49, ***55, 56, 57*** 4.8 LDE Wronskian #9, 13, 17, 23, 27, 29, 31, 35, 39

Date	Discussion	Assessment	Suggested Exercises
Unit 3 Inner-Product Spaces			
Wk 8 3/3 3/5 3/7 Wk 9 3/10 3/12 3/14 Wk 10 3/17 3/19 3/21 3/24 3/26 3/28	§ 5.1-5.5 (start § 6.1-6.5) **Spring** **Break** **--------**	 **Quiz 6 (5.1)** **Quiz 7 (IPs and Ortho Bases)** **Test 3 & Portfolio** No classes this week 3/24-28 --------	5.1 #5, 7, 9, 11, 13, 15, 21, 25, 29, 33, 37, 41, 45, 47, 53, 57, 61, 65, ***73, 74*** 5.2 #5, 13, 17, 23, 27, 29, 31, 33, 37, 41, 45, 49, 51, 59, 65, 75, 81***, 85, 86*** 5.3 #5, 11, 13, 19, 23, 29, 39, ***55, 56, 67*** 5.4 Orthogonal Subspaces #7, 9, 11, 19, 21, ***43, 44*** 5.5 Cross Products #7, 11, 13, 15, 19, 23, 35 Geometry #***45, 47, 49, 62***
Unit 4 Linear Transformations, Eigenvalues, and Eigenvectors			
Wk 11 3/31 4/2 4/4 Wk 12 4/7 4/9 4/11	§ 6.1-6.5 **AACS**	 **Project 2 Report** **Quiz 8** no class Wednesday	6.1 #3, 15, 19, 21, 23, 25, 29, 31, 33, 39, 49, 53, ***65, 67, 68*** 6.2 #1, 3, 5, 9, 11, 15, 17, 19, 23, 27, 31, 33-36 all, 41-44 all, ***49, 51, 53, 65, 66*** 6.3 #1, 5, 7, 9, 11, 15, 19, 23, 25, 27, 33, 35, 41, 43, ***52, 53, 54*** 6.4 #3, 7, 13, 15, 17, 21, ***25, 41, 42*** 6.5 (TBD) Shears and Rotations
Wk 13 4/14 4/16 4/18 4/21 4/23 4/25 4/28 4/30 5/2	§ 7.1-7.3 	 **Quiz 9** **Test 4 & Portfolio** **Quiz 10 (Equiv Conditions)** *Presentations 4/25-5/2 **Project 3 Report** Presentations end	7.1 #1, 3, 5, 9, 11, 15, 19, 23, 27, 33, 51, 53, ***59, 67, 68***, 69, 71, ***79, 80*** 7.2 #3, 7, 9, 11, 15, 19, 23, 27, 29, ***37, 38*** 7.3 #9, 13, 17, 23, 27, 35, 41, 45, 49, 51, ***53, 54*** 7.4 (TBD) Rotations of Conics (TBD) Constrained Optimization
5/7		**Final Exam** Wed 5/7 3:30-4:40	Final exam according to BJU final exam schedule

Project 1 Problem Sets:

Skew Symmetric Matrices § 2.2 #71, 72, 73, 74, 75, 76, 77, 78; § 2.3 #67
Idempotent Matrices § 2.3 #66, 70, 80; § 2.4 #54, 55
Nilpotent Matrices Textbook Chapter 2 Project all problems
Special Determinants § 3.3 #65, 69, 70, 83

Project 2 Options: Least Squares Approximations (5.4 and 5.5) or Fourier Approximations (5.5)
or (for CpS majors with permission) Textbook Chapter 5 QR-Factorization Project with programming

Project 3 Options: Rotations of Conics (7.4) or Constrained Optimization (7.4)