

Ma 421
Modern Geometry
Fall/2024

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Office Hours:	MWF 8:00 – 8:50 a.m. Tu 1:30 – 2:45 p.m. Th 8:00 – 9:15 a.m. Please email or text to confirm availability
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Communication Policy:	Feel free to email or contact me via Microsoft Teams for questions and/or extended help. You may text where appropriate (not during class).
Classroom:	AL 301
Meeting:	MWF 9:00 - 9:50 a.m.
Credit/Load:	3/3
Textbook(s):	<ul style="list-style-type: none"> • <i>Modern Geometries</i> by James R. Smart, Fifth Edition ISBN 0534-35188-3 • <i>Episodes in Nineteenth and Twentieth Century Euclidean Geometry</i> by Ross Honsberger, ISBN 0883856395; This text is optional. • Paper: Plimpton 322 is Babylonian exact sexagesimal trigonometry • The Geometer's Sketchpad web version; This text is optional. • Non-Euclid: Interactive Javascript Software

Catalog Description:

Methods and theory of transformational geometry in the plane and 3-space, finite geometry, advanced Euclidean Geometry, constructions, non-Euclidean geometry, and projective geometry, and *Geometer's Sketchpad* experiences.

Course Context:

Modern Geometry is required for Math Education majors. Most, if not all, of this course's objectives are aligned with the NCTM standards for Math Ed students. Additionally, as an elective course in the Math program, it fulfils the following program goals.

Mathematics Program Goal
MM1. Graduates will exhibit maturity in the development and implementation of mathematical procedures.
MM2. Exhibit independent and abstract thought and make judgments about the value of innovative developments from a Biblical world view.
MM3. Display understanding of what constitutes mathematics, including its role within the framework of Biblical Truth.
MM4. Provide a solid foundation for graduate studies in mathematics.

Course Goals:

1. To develop mathematical maturity and independent thinking.
2. To develop a greater interest in exploring geometrical ideas independent of the teacher—especially in the context of non Euclidean geometry.
3. To develop the student's abilities to prove or justify geometrical ideas.
4. To develop competence in solving problems in the context of transformational geometry, using synthetic geometry, analytical geometry and linear algebra tools.
5. To expand the student's Euclidean Geometry base by exploring more advanced topics and by broadening the students experiences beyond the familiar topics.
6. To develop geometric understanding through advanced constructions and justification of classical Greek impossible constructions.
7. To broaden the student's background in mathematical history.
8. To enrich the prospective teacher's background and understanding of geometry for future teaching.

Course Objectives:

The student will be able to

1. Make logical connections between Modern Geometry topics and the ideas of function, analytic geometry, linear algebra, abstract algebra, direction cosines and vectors. (NCTM 2.5, 2.6) *Measured in both chapter 2 tests.*
2. Identify and characterize symmetry groups of transformations in the context of geometric figures and the applicable isometries. For a given figure identify all possible symmetry operations and justify that they form a group under function composition. (NCTM 2.6, 3.2) *Measured in symmetry quiz, subsequent test, and final exam.*

3. Express the ideas of transformational geometry using both synthetic methods and analytic geometry methods. For example, given the linear equation of a line the student can derive a matrix equation for a reflection in that line. (NCTM 1.4, 2.5, 3.2, 3.9) *Measured in chapter test and final exam.*
4. Solve problems using transformational geometry. For example, if given three distinct parallel lines (not equally spaced) the student should be able to use an isometry like rotation to find an equilateral triangle with one vertex on each of the lines. (NCTM 3.2, 3.3) *Measured in transformation paper and final exam.*
5. Identify and locate many points, segments, and circles associated with a triangle. For example, the student must be able to prove the existence of the 9 point circle and the location of its center at the center of the Euler line. (NCTM 3.1, 3.8) *Measured in Sketchpad test and final exam.*
6. Prove the theorems of Menelaus and Ceva and use them to establish the collinearity of points and the concurrency of segments (e.g. medians intersect at the centroid). (NCTM A.3.8, 3.9) *Measured in chapter test.*
7. Show a competence in all the basic construction techniques of high school geometry and use those skills to construct more advanced figures related to II-6 above. The student must be able to justify the construction by giving the basic reasons (or crux) why it is true. (NCTM A.3.1, 3.3, 3.8) *Measured in constructions paper and final exam.*
8. Show a competence in various compass-based construction techniques including rusty compasses and compass only constructions. Additionally, the student will be able to discuss the history of this class of constructions. (NCTM 3.8, 3.10) *Measured in final exam.*
9. Summarize the key ideas presented in the axioms of Projective Geometry; give the basic definitions like range, pencil, perspective, projective, etc. Construct the fourth point of a harmonic sequence or the fourth line of a harmonic pencil. (NCTM A.3.1, 3.8) *Measured in final exam.*
10. Survey the history and show an understanding of the geometric structure of non-Euclidean geometry, including the undefined terms, defined terms, axioms, and resulting theorems. Students should know the theoretical and practical consequences of choosing different parallel postulates. Students will use the characteristic postulate of hyperbolic geometry and resulting properties to show that the sum of the angles of a triangle is less than 180. (NCTM A3.1, 3.8, 3.10) *Measured in non-Euclidean paper and final exam.*
11. Using the software, Geometer's Sketchpad, find the images of figures under the isometries, construct the special points associated with a triangle like orthocenter, 9-point center, and circumcenter. Produce orthogonal circles using a harmonic range, and generate fractal curves like the snowflake curve or the Sierpinski triangle. (A.3.1, 3.8) *Measured in Sketchpad activities.*
12. Read 200 pages of mathematics history both in non-Euclidean geometry, Euclidean geometry and other fields. (NCTM A.3.10) *Self-reported.*

Course Content:

A. Transformational Geometry

1. Mappings, transformations, and function composition
2. Groups of transformations and types of groups
3. Orthogonal matrices, orientation, types of isometries, and matrix equations for transformations
4. 3 space isometries, similarities, and dilations
5. Applications of transformational geometry to problem solving without analytic geometry

B. Advanced Euclidean Geometry

1. Circles, points and segments related to a triangle
2. Heron's formula for the area of a triangle
3. Theorems of Menelaus and Ceva, ratios of division
4. Nine point circle, Euler line, Miquel point
5. Golden ratio, Golden rectangle, Golden spiral, Fibonacci sequence

C. Constructions

1. Review of methods, collapsing compass constructions, constructible numbers, constructions related to the Golden ratio.
2. Advanced constructions, datum, classical Greek impossible construction – proofs
3. Using Geometer's Sketchpad to construct geometric figures and study their properties

D. Projective Geometry

1. Definitions, axioms, Desargue's theorem, duality
2. Harmonic sequence and harmonic pencil, proof of uniqueness of fourth set, given the other three
3. Projective generation of conics, line and point conics

E. Non Euclidean Geometry (Self study project)

1. Ideas of hyperbolic geometry, ideal points and omega triangles
2. Saccheri and Lambert quadrilaterals
3. Limiting curve, equidistant curve, elliptic geometry, models

Class Policies:

Homework

Homework is crucial to success in this course. It is also one of the primary means by which you represent yourself as a "professional" in academia, and the way in which you will develop the mathematical habits that will help you be successful on the larger quizzes and tests. Therefore we have the following expectations for homework, and failure to meet these expectations may mean that your assignment will be

returned to you to be corrected.

You are responsible for checking all of your homework problems from the answers in the back of the book. Complete Solutions Guides are available on reserve in the library at the check-out desk. You should use them frequently. Do not copy the answers but use the Guides as a resource to learn the material that you are expected to know. Place the number you got correct (on your initial attempt) out of the total at the top of the page. Any problems that are incorrect on your initial attempt, correct in your homework to the side of your original work. Missing these problems will not cause your grade to be lowered but will give an idea of your areas of weakness in understanding the material.

Class Deportment

Compliance with student handbook policies is expected during class. All class deportment should reflect your intention to pay attention, to be polite, and to be professional. Laptops may be used to take notes and to perform calculations and constructions during class. Please do not use the laptop for other purposes during class since studies have demonstrated that one's student's misuse of a laptop during class tends to diminish the learning of the surrounding students.

Accommodations for Students with Disabilities

Any student with disabilities or any additional needs is encouraged to contact the instructor within the first week of the course to discuss accommodations that may be necessary.

Attendance Policies and Academic Penalty for Absences

- Attendance will be tracked and reported according to the university attendance policy:
 - Students are expected to attend and arrive on time for all scheduled class sessions, including the final exam.
 - Students are to use effective time management in order to meet their class attendance responsibilities.
 - Up to three (3) personal Absences may be taken for funerals, for sickness, for doctor's or dentist's appointments, for visits and interviews at graduate schools or for interviews for future employment.
 - Up to four (4) Service Absences may be taken to attend approved academic functions or conferences, approved Christian service projects, required military duty or as part of an intercollegiate athletic team. However, students who exceed the Personal Absence limit due to a chronic illness are not eligible to participate in events that require Services Absences. Also, students who are on any type of academic restriction (including probation) or who have a current grade report with a cumulative GPA below 2.0 are not eligible to participate in events that require Service Absences.
 - Arriving late or leaving early is marked as a partial attendance. Three (3) partial attendance marks count as a personal absence.
 - Missing more than 15 minutes of class is marked as an absence.
 - For more details and information about chronic illness, please see the [university attendance policy](#).
- Students are responsible for all material and announcements given in class.
- If a student is absent for an exam and has a good reason, the student is to notify the instructor before the exam is covered in the next class.

Late Work

Work is due at the specified deadline. Work submitted late will incur a late penalty up to 25% off for work submitted within a week of the due date. Notify the instructor immediately if a situation arises necessitating an extension. Early, impressive work is encouraged and may be rewarded.

Academic Honesty

You are expected to uphold the school standard of conduct relating to academic honesty: <http://home.bju.edu/academics/integrity.pdf> You must assume full responsibility for the content and integrity of the academic work you submit. The guiding principle of academic integrity is that your submitted work; examinations, reports, and projects must be your own work. You are guilty of violating this policy if you:

- Represent the work of others as your own.
- Use or obtain unauthorized assistance in any academic work.
- Give unauthorized assistance to other students.
- Modify, without instructor approval, an examination, paper, record, or report for the purpose of obtaining additional credit.
- Misrepresent the content of submitted work.

Misrepresenting your work is unethical in any setting. In an academic setting, it is a breach of the university policies. The penalty for cheating is severe. Any student cheating is subject to receive a failing grade for the assignment and will be reported to the Dean. If you are unclear about whether a particular situation may constitute cheating, consult with your instructor about the situation. For this class, it is permissible to assist classmates in general discussions of construction techniques. General advice and interaction are encouraged. Each of you must develop your own solutions to the assigned projects, assignments, and tasks. In other words, you may not "work together" on graded assignments with other students unless instructed to work as a group on a particular assignment. Such collaboration constitutes cheating. You may not use or copy (by any means) another's work (or portions of it) and represent it as your own.

Learning how to use sources appropriately is a vital part of your development as a student. To assist you in this endeavor, the university uses Turnitin, an academic plagiarism checker. Registration in this course constitutes permission for the teacher to submit any or all assignments to Turnitin.

Web / AI Resources

Any use of web or AI resources should be documented. For the Non-Euclidean Geometry project, the rubric specifies scoring adjustments based upon using these tools.

Need Help?

You must seek help when needed because you are the only one who knows when you need it. If you need help, reach out to one of the following ways:

- Teacher – It is always best to seek help in person, either in my office or before class, if time allows. You may also text me or email me in order to set up a time in which to come see me if you have a class or are working during my announced office hours. My door is always open during my office hours. I encourage you to come see me for help.
- Classmates – Studying for tests and quizzes with another Ma407 student is helpful. Unless an assignment is a group assignment, it may not be done in collaboration with anyone else. No group assignments are intended to be assigned this semester.

Instructional Resources:

- A. Textbooks: Modern Geometries, 5th ed. by James R. Smart
- B. Resource Textbooks: (For self-study paper)
 1. Introduction to Non-Euclidean Geometry, 2nd ed. by David Gans
 2. Introduction to Non-Euclidean Geometry, by Harold Wolfe
 3. Non-Euclidean Geometry, by H. Meschkowski
 4. Non-Euclidean Geometry, by Roberto Bonilla
 5. Roads to Geometry, by Wallace and West
 6. Elementary Geometry from an Advanced Standpoint, by Edwin Moise
 7. Introduction to The Foundations & Fundamentals, by Eves and Newsom (The next two are for Geometer's Sketchpad Activities)
 8. Active Geometry, by David A. Thomas
 9. Exploring Geometry, by Key Curriculum Press
- C. Magazines, Journals and Articles
 1. The Mathematics Teacher - NCTM
 2. American Mathematics Monthly, Vol. 97, #8 Geometry Issue
 3. "Philosophical and Mathematical Considerations of a Finite Universe" - Jerome Lewis

Course Requirements:

- A. Quizzes: Symmetry, 2D and 3D isometries, duality
- B. Three Tests
- C. Non Euclidean Geometry Paper
- D. Transformations Application Paper
- E. Advanced Constructions Paper
- F. Homework Problem Sets
- G. Electronic Drawing Activities
- H. 200 pages of Mathematics History reading
- I. Cumulative Final Exam

Evaluation:

A. Quizzes	39	<i>Grading Scale</i>
B. 3 Tests	230	A: 90 – 100
C. 3 Papers	225	B: 80 – 89
D. Homework Total	100	C: 70 – 79
E. 8 Electronic Drawing Activities	86	D: 65 – 69
F. Final Exam	150	F: < 65
Total	825	

- G. 200 or more pages of History of Math reading are required as part of the course. There is no write-up required and no points are assigned to this requirement, but you cannot pass the course without it. History reading related to the non-Euclidean geometry paper can count toward the page total. As you read from various sources keep a record of the source and the number of pages. This record can then be turned in at the final exam and will not require a lot of work remembering the sources or pages.

Copyright Policy:

Copyright-2017 (Knisely) as to this syllabus and all lectures. (Some class policies have been adapted from those policies in use in the math department.) Students are prohibited from selling (or being paid for taking) notes during the course to, or by any person, or commercial firm, without the express written permission of the professor teaching the course.

Additional Documents

- [Lecture schedule](#)
- [Sketchpad documents](#)
- [Assignments](#)