

Ma 435
**Analysis of
Metric Spaces**
Fall/2025

College of Arts and Science
Division of Mathematical Sciences

Instructor:	James A. Knisely, Ph.D.
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Office Hours:	MWF 7:45 – 8:50 a.m. Tu 1:30 – 2:45 p.m. (electronic) Th 7:45 – 9:15 a.m. Please email or text to confirm availability
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Communication Policy:	Feel free to email or contact me via Microsoft Teams for questions and/or extended help. You may text where appropriate (not during class).
Classroom/Meets:	AL 314 / MWF 9:00 - 9:50 a.m.
Credit/Load:	3/3
Textbook(s):	<i>Introductory Functional Analysis with Applications</i> 1st Edition by Erwin Kreyszig (Author) Other editions may be used. Please check with professor.

Catalog Description:

The real and complex number systems, point-set theory, concepts of limits and continuity, differentiation of functions of one and more variables, functions of bounded variation, rectifiable curves and connected sets. Required calculator: TI 89 or Nspire CAS. Prerequisite(s): Ma 235, Ma 299, Ma 311. 3 Credits.

Course Context:

Analysis of Metric Spaces is a course primarily taken by mathematics majors. Its presence in the mathematics program is intended to give the students exposure to calculus-related concepts in a more general setting. As a 400-level course, students will be expected to demonstrate the ability to produce, not simply replicate, proofs. Additionally presenting ideas, such as proofs, to peers will be expected.

Course Goals:

1. To develop a Christian perspective of calculus and related scientific endeavor
2. To develop mathematical maturity and independent thinking
3. To develop a greater appreciation for the beauty and power of abstract reasoning
4. To develop the ability to express mathematical concepts both verbally and in written form

Course Objectives:

With at least 70% accuracy, the student will be able to do the following:

1. Write the definitions of all terms and concepts using complete sentences. (Not simply describing ideas associated with a concept).
2. Provide examples of entities which have/don't a property. For example, a metric space that is not a normed space.
3. Write the proofs of classic theorems such as Riesz's lemma.
4. Discuss and/or demonstrate an algorithm or method such as the completion of a metric space and the Gram-Schmidt orthogonalization method. Apply proof methods to analysis problems as demonstrated in the solution of homework problems.
5. Present solutions to homework problems (to the class).

Course Content:

Chapter 1: Metric Spaces: examples, convergence, completeness, and completing a metric space.

Chapter 2: Normed and Banach Spaces: vector spaces, finite dimensional normed spaces, linear operators, bounded and continuous linear operators, linear functionals, linear operators and functionals on finite dimensional spaces

Chapter 3: Inner product and Hilbert Spaces: inner product space, orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets, functionals in Hilbert spaces

Chapter 4: Fundamental Theorems for Normed and Banach Spaces including Hahn-Banach, Category, Uniform Boundedness, and the Open Mapping Theorem

Chapter 5: Banach Fixed Point Theorem and applications to root finding, systems of linear equations and differential equations

Course Evaluation:

All course/assignment grades are based on the evaluation of the work communicated by the student. Unclear or incomplete communication of the solutions, which includes the process, may result in a penalty at the professor's discretion.

Letter grades will be based on a standard 10-point scale. **Note:** *The rounding of grades is not guaranteed* and will be dependent, in part, on student behaviors such as the timeliness of submissions, class attendance, and participation in class.

Projected Points:

Category	Points
Quiz	295
Final	75
HW	123
Present	35
Total	528

Course Requirements:

- There will be six (substantial) quizzes covering from 2 to 6 sections from the text.
- The final exam will contain a cumulative element but will focus primarily on the content since the last quiz.
- The [assigned problems](#) for homework are on a [separate page](#).
- For each homework section, the student will write a solution to a problem on the board and explain it to the class. The homework assignments along with presentations of the homework will be worth roughly 30% of the overall grade.

General Policies:

Department

Compliance with student handbook policies is expected during class. The classroom is to be a professional environment. That means you are to come to class prepared for the day's discussion, your attention is expected to be on course related material, and you are expected to positively contribute to the class.

Emergencies During Class

In case of emergency requiring evacuation, students will go down the stairs on the fountain side and exit the door facing Wade Hampton underneath the stairs. Students will immediately cross the street and gather by the fence with their class. If we are unable to exit the building, the professor will instruct the students on the best course of action. To be able to respond quickly to external threats, professors may keep classroom doors locked. If you are late arriving to class, you may need to knock on the door and be let in.

Absences

BJU attendance policy is in effect (see <https://home.bju.edu/bju-policies/> for details).

- Scheduled tests/quizzes should be taken before your planned absence; please contact your professor to make arrangements for doing so. You are personally responsible for getting notes from your classmates and discussing the missed material with them. You should not expect your professor to privately re-teach you the material you missed. Your professor is always available to help you with specific questions. If an unannounced quiz/assessment is taken during the class that you miss, you will NOT be allowed to make it up, and you WILL receive a zero on the assignment. Work may always be completed early (see your professor if you wish to take a test early).
- Missing an in-class test because you feel you are not prepared to take it is not acceptable. Work missed for this reason will not be made up and you will receive a zero on the assignment.
- For absences due to incapacitating illness or emergency, you should contact the instructor as soon as you realize you will not be in class to make arrangements to make up any missed work. Tests will be made up without penalty for the first occurrence. Each subsequent time a test is missed because of incapacitating illness or emergency, an additional 10% grade penalty for that test will be incurred. A 10% penalty will be assessed for a late submission of take-home tests. All late work must be made up by the next class period unless other arrangements have been made with the professor.

Presentation of Work

The goal is professional, fluent, and clear communication of what you know.

PW 1: Proper use of mathematical notation is expected. The structure of notation conveys specific meaning and should be used appropriately.

PW 2: Mathematical presentation is like grammar. There are subjects, verbs ($=$, \leq , $>$, etc.), and objects. Always write in complete sentences.

PW 3: Tests/presentations/projects are not only about what you know, but about what you can communicate about what you know so the presentation of your work/logic should always be neat, orderly, clearly defined, and with the appropriate amount of supporting detail. (Excessive steps are not required; however, answers alone are not (usually) acceptable.)

PW 4: Always work down the page. (Working in multiple columns is generally not acceptable.) There should be one problem worked in each row because this contributes to clarity and the development of your logical argument.

PW 5: Skip lines between problems. If you have dense handwriting, skip every other line and skip 2-3 lines between problems.

PW 6: Clearly label problems/sub-problems. Problems do not necessarily have to be worked in order but must be clearly labeled either way. Your professor will communicate their expectation on presenting problems out of order.

PW 7: Answers are to be presented as the logical conclusion of your work, not as the only important thing (e.g. at the start of the problem and/or unconnected with any justifying work).

PW 8: Work should be submitted on clean 8.5 × 11 inch (standard-size) paper and should not be submitted with spiral/ripped edges.

PW 9: Take-home tests (when time is not limited) should be neatly presented (rewritten, organized, no scratchwork, etc.) as a final polished piece.

Your professor may refuse to accept work that does not meet the minimum presentation requirements above, or they may choose to deduct up to 10% from the assignment.

Problems Expectations

The goal is to prove your mastery (not your just barely comprehending). Failure to meet these expectations will be reflected in lower test scores.

PE 1: Read all words carefully in a question. Everything is important, so know what the meanings of all words are and how those words tell you to respond.

PE 2: Theory is a precise expression of important ideas. While it is not graded word for word, jot for jot, the precise ideas must be maintained. Embrace thorough, smooth learning and presentation. Can you recite the theory from the last class period quickly, comfortably, and conversationally?

PE 3: Theory tells us how to solve problems. Know exactly what problems connected to each theorem or definition look like, and know how to solve them.

PE 4: Know what the key steps of each problem are. Present only the key steps (or the minimum needed to get the answer right and show all your logic).

PE 5: Do enough practice for each type of problem so that you are smooth.

Homework

Homework is intended as a space for you to develop conceptual understanding and skill at communicating your understanding. It is collected only as way to help you develop discipline and maturity. It is due at the start of the indicated class period (or may be turned in early). No late homework is accepted, and you may not copy solutions from another source.

You are done with homework when you can anticipate test questions, solve them, and appropriately communicate those solutions regardless of how many or few problems you have completed.

Late Policy

Assignments not submitted as directed by the due date will incur the following late penalty.

- No late homework/in-class assignments are accepted.
- Written assignments/projects/take-home tests are penalized at 10% per day for the first three days and a grade of 0% after that. Oral presentations are a 0 if not presented on the day assigned. Late paper submissions must include the date and time the paper is submitted and be in the credenza by 8am the next day. The next day penalty begins at 8am.
- In-class tests must be taken by the date given in class (or selected time in the case of an oral exam) unless there is incapacitating illness (see attendance policy below). Missing a test/taking the test late (including an oral exam) will result in a 10% penalty unless excused by the professor. Tests should be made up prior to the next class period unless other arrangements have been made with the professor.
- Work may always be completed early. Contact your professor if you wish to take a test early.

Academic Integrity Policies:

The university's Academic Integrity Policy is in effect (see <https://home.bju.edu/bju-policies/> for additional details).

Definitions of Integrity Violations

Integrity is the reflection of the character and nature of God in our actions; therefore, students will be expected to work with integrity. In academia, violations of integrity generally fall into one or more of the following categories:

- Cheating: unauthorized use or attempted use of assistance, information, or aids in any academic assignment
 - Falsification: submitting work done by others, changing work after submitting an assignment, reporting false information about the completion of an assignment
 - Unacceptable collaboration: working with others when not permitted, using AI to generate ideas, thoughts, or content without the explicit permission of the professor
 - Facilitation of Cheating: helping another student violate academic integrity, communicating quiz/test questions to other students
 - Plagiarism: the intentional or unintentional use to any degree of the ideas or words of one's source material without proper acknowledgement
- All work done for this class must represent your own effort, your own understanding, and your own communication of the material.

Course Integrity Policies

If information is taken from other sources (which is at times appropriate), it always needs to be referenced and credit given where it is due. Use standard referencing techniques as taught in En 102. Solutions found on the internet are not to be copied.

- Homework: While you are encouraged to work together on the homework assignments, simply copying someone else's solution is neither useful nor acceptable. Your homework should represent your work and your understanding of the work.
- Tests (In-Class and Take-Home): No resources may be used while taking the test unless permitted by the professor. The presence of any unauthorized material on your desk, in your calculator, on your laptop, etc. while taking a test will be construed as cheating and will be dealt with as such.
Internet/AI enabled devices or any communication devices (including but not limited to smart glasses, watches, earbuds, etc.) are not permitted to be used and should be stored out of sight during the testing period. Access these type of devices during the test will be construed as cheating and will be dealt with as such.
Cheating on a test will likely result in a zero on the test and will be submitted to the Academic Integrity Committee.
- Projects: You are encouraged to discuss the general ideas needed to complete the project as discussed in this course with your classmates but are not permitted to work together on your project (outside of your own team and any faculty appointed advisors). Your projects must represent your own ideas, your own work, and your own communication of your work.
Assignment submissions will be evaluated for plagiarism and AI usage at the discretion of the professor. If you have a question about any source you are considering using, it is wise to gain your professor's approval before using it. You are always permitted to ask your professor for help. Any help they choose to provide is acceptable.

AI Usage Policy

The goal of the assignments in this course is to learn to develop the skills covered, NOT to complete the tasks assigned. The use of AI to complete or jumpstart tasks defeats the goal of the assignments. Therefore, you may not use generative AI tools in this course for any assignment without the professor's express permission. AI tools include, but are not limited to, CoPilot, Apple Intelligence, Chat GPT, Bing Chat, Google Bard, Grok, Deepseek, Grammarly, and language translators.

Use of generative AI to develop code (such as Python or R) may be helpful during the project (each student has permission to use AI for only this purpose, other purposes require express permission). It would be wise to consult with your professor before incorporating it into your work. Reliance on AI to generate code has not yet resulted in an acceptable paper. If you do use it, you must document it as indicated above. You may NOT use AI to generate the text/discussion in your project.

Documentation of Permitted AI Use

Should an AI tool be used with permission, its use must be documented (including the tool used, a summary of the prompts provided and the portions of the assignment that were based on AI generated work). See <https://style.mla.org/citing-generative-ai/> for details on citing the use of AI.