Ma 308 ~ Theory of Interest

Homework Problems

General Annuities with "Off" Payments

page 159, # 20. You are given: the present value of a $6n$-year annuity-immediate of 1 at the end of every year is 9.996; the present value of a $6n$-year annuity-immediate of 1 at the end of every second year is 4.760; the present value of a $6n$-year annuity-immediate of 1 at the end of every third year is X. Calculate X. [SOA 5/90 #9]

page 159, # 23. Which of the following statements are true? [CAS 5/87 #5]

1. $s_{\overline{n}|} - a_{\overline{n}|} = i a_{\overline{n}|} s_{\overline{n}|}$ 2. $\ddot{s}^{(m)}_{\overline{n}|i} - s^{(m)}_{\overline{n}|i} = \frac{i^{(m)}}{m} s^{(m)}_{\overline{n}|i}$ 3. ${}_{1/4}|\ddot{a}^{(2)}_{\overline{n}|i} + a^{(2)}_{\overline{n}|i} = a^{(4)}_{\overline{n}|i}$

page 160, # 27. Janis needs an amount on January 1, 2025 to provide for a lump sum of 50,000 and a 15-year annuity-due with semiannual payments of K. The amount will be accumulated by 25 annual deposits of K beginning on January 1, 2000. The deposits accumulate at a nominal rate of 4% compounded semiannually. The annuity payout is based on a nominal rate of 3% compounded semiannually. Determine an expression for K. [SOA 5/87 #9]

(A) $\dfrac{50,000}{\frac{s_{\overline{50}|0.02}}{a_{\overline{2}|0.02}} - \ddot{a}_{\overline{30}|0.015}}$ (B) $\dfrac{50,000}{\frac{s_{\overline{50}|0.02}}{s_{\overline{2}|0.02}} - \ddot{a}_{\overline{30}|0.015}}$ (C) $\dfrac{50,000}{\frac{s_{\overline{50}|0.02}}{s_{\overline{2}|0.02}} - \frac{a_{\overline{30}|0.015}}{a_{\overline{2}|0.015}}}$

(D) $\dfrac{50,000}{\frac{s_{\overline{50}|0.02}}{a_{\overline{2}|0.02}} - \frac{a_{\overline{30}|0.015}}{s_{\overline{2}|0.015}}}$ (E) $\dfrac{50,000}{\frac{s_{\overline{50}|0.02}}{a_{\overline{2}|0.02}} - a_{\overline{30}|0.015}}$

page 160, # 29. You anticipate having to pay \$30,000 per year for your child's college education starting 10 years from now. You plan to finance four years of college by making quarterly deposits in a savings account starting now. The final deposit is made three months prior to the first college payment, for a total of 40 deposits. Each annual college payment is made in full at the beginning of the school year. If the savings account earns 8% per annum convertible quarterly, what should your quarterly deposit be? [CAS 5/86 #6]

page 162, # 39. A perpetuity paying \$50 on the last day of each year was purchased on January 1, 1928. On January 1, 1978, the perpetuity was exchanged for a 15-year annuity-due with semiannual payments of amount X. The interest rate is 6%, convertible monthly. Find X. [SOA Sample/84 # 16]

(A) $\$10,000 \dfrac{a_{\overline{6}|0.005}}{a_{\overline{180}|0.005} a_{\overline{12}|0.005}}$ (B) $\$10,000 \dfrac{a_{\overline{6}|0.005}}{a_{\overline{180}|0.005} \ddot{a}_{\overline{12}|0.005}}$ (C) $\$10,000 \dfrac{a_{\overline{6}|0.005}}{a_{\overline{180}|0.005} s_{\overline{12}|0.005}}$

(D) $\$10,000 \dfrac{\ddot{a}_{\overline{6}|0.005}}{a_{\overline{180}|0.005} a_{\overline{12}|0.005}}$ (E) $\$10,000 \dfrac{\ddot{a}_{\overline{6}|0.005}}{a_{\overline{180}|0.005} s_{\overline{12}|0.005}}$

The previous problems are taken from: Harold Cherry and Rick Gorvett. *Study Manual for Exam FM/Exam 2: Financial Mathematics and Financial Economics.* Actuarial Study Materials, 7th edition, 2008.