Using the TA Completion Report, create a plan to complete this assignment by working regularly throughout the semester. Submit a copy of your plan in class on 9/3/24.

For each proof, you should start with a statement like "Given _______, show _______," then present your argument (and at no time claim what you are trying to prove ... so avoid "two-sided" proofs.)

A typed paper is not required, but neat/proper presentation is mandatory. Failure to do so will result in up to 1 point penalty per problem.

This assignment will be worth 150 points (4 pts per problem/subproblem, 10 pts for completing on schedule, penalty for poor presentation or failure to submit a filled out and signed Completion Report). Assignments must include a completed integrity form.

1 Moving Money - Chapters 1-2

Theory Assignment 1. *Consider the relationship between compound interest and compound discount using $P = S\,(1-d)^n$ and $P = S(1+i)^{-n}$.*

a) *Develop a relationship to convert an interest rate compounded m times a year to a discount rate compounded p times a year. State the relationship when $m = p = 1$ (aka state the relationship to convert i to d).*

b) *From first principles, prove $d = i\nu$*

c) *From first principles, prove $1 - \nu = d$*

d) *From first principles, prove $\nu = 1 - d$*

e) *From first principles, prove $i - d = id$*

Theory Assignment 2. *In terms of accumulation functions, the condition that there should never be an advantage or disadvantage to closing and immediately reopening one's account means that for all positive real numbers s and t, $a(s+t) = a(s)a(t)$. Assume that $a(t)$ is differentiable for all $t > 0$ and differentiable from the right at $t = 0$. These conditions on derivatives amount to the accumulation function being continuous and not having any sudden changes in direction. Show that these conditions imply that $a(t) = (1+i)^t$ by showing the following:*

a) $a'(s) = a(s) \lim\limits_{h\to 0} \dfrac{a(h)-1}{h}$

b) $a'(s) = a(s)a'(0)$, *where $a'(0)$ is the right-handed derivative*

c) $\int\limits_0^t \dfrac{d}{ds} \ln a(s) ds = a'(0)t$

d) $\ln a(t) = a'(0)t$

e) $a'(0) = \ln(1+i)$

f) $a(t) = (1+i)^t$

Theory Assignment 3. *Let m be a positive real number. Suppose interest is paid once every m years at a nominal interest rate $i\left(\frac{1}{m}\right)$.*

a) *State the effective interest rate per $m-$year period.*

b) *Find an expression for $i\left(\frac{1}{m}\right)$ in terms of i.*

c) *Define $d\left(\frac{1}{m}\right)$ to be the nominal discount rate payable once every m years. Find a formula that gives $d\left(\frac{1}{m}\right)$ in terms of $i\left(\frac{1}{m}\right)$, and a formula that gives $d\left(\frac{1}{m}\right)$ in terms of d.*

Theory Assignment 4. *Let $i(x) = x\left[(1+i)^{\frac{1}{x}} - 1\right]$ for $x > 1$.*

a) *Show that $\ln\left[(1+i)^{\frac{1}{x}}\left(1 - \frac{\ln(1+i)}{x}\right)\right] < 0$ is a sufficient condition to demonstrate that $i(x)$ is a decreasing function.*

b) *Let $z = \frac{\ln(1+i)}{x}$ and show $1 - z < e^{-z}$ is a sufficient condition to demonstrate that $i(x)$ is decreasing.*

c) *Show that $1 - z < e^{-z}$ for all $x > 1$.*

d) *Argue that $i(p) < i(m)$ for all $1 < m < p$.*

Theory Assignment 5. *If $m > 1$, place the five measures i, $i(m)$, d, $d(m)$, and δ in order smallest to largest and justify your answer.*

Theory Assignment 6. *Given Fund I grows according to simple interest at rate r, and Fund D grows according to simple discount at rate s.*

a) *Find the force of interest δ_t^I acting on fund I at time t, and the force of interest δ_t^D acting on fund D at time t.*

b) *Find all t such that $\delta_t^I = \delta_t^D$, and state the condition under which this is valid.*

Theory Assignment 7. *Derive a "rule of n's" that uses only addition, subtraction, multiplication, and division to approximate the length of time it takes for money to triple. Chose your rule so that it gives an especially good estimate for 8%. After you have stated your rule, compare the approximations it gives for rates of 4% and 12%. Quantify the quality of these estimations.*

2 Annuities - Chapters 3-4

Theory Assignment 8. *Using both algebraic and cash flow techniques, prove*

a) $\ddot{a}_{\overline{n}|i} = 1 + a_{\overline{n-1}|i}$

b) $\ddot{s}_{\overline{n}|i} = s_{\overline{n+1}|i} - 1 = s_{\overline{n}|i} - 1 + (1+i)^n$

c) $a_{\overline{10}|}(1+i)^6 = \ddot{s}_{\overline{5}|} + \ddot{a}_{\overline{5}|}$.

d) *For* $n > m$, $a_{\overline{n}|}(1+i)^m = s_{\overline{m}|} + a_{\overline{n-m}|}$

e) $\dfrac{\ddot{a}^{(m)}_{\overline{n}|}}{\ddot{a}^{(m)}_{\overline{p}|}} = \dfrac{a^{(m)}_{\overline{n}|}}{a^{(m)}_{\overline{p}|}} = \dfrac{a_{\overline{n}|}}{a_{\overline{p}|}} = \dfrac{1-\nu^n}{1-\nu^p}$

f) ${}_{m|}a_{\overline{\infty}|} = \dfrac{1}{i} - a_{\overline{m}|i}$

g) $\ddot{a}_{\overline{\infty}|} = \dfrac{1+i}{i}$

Theory Assignment 9. *Consider the sequence of payments* $1, 2, 3, \ldots, n-1, n, n-1, n-2, \ldots, 3, 2, 1$, *and let PV represent its present value. Using both algebraic and cash flow techniques, prove* $PV = (Ia)_{\overline{n}|} + \nu^n (Da)_{\overline{n-1}|} = a_{\overline{n}|}\ddot{a}_{\overline{n}|}$.

Theory Assignment 10. *A perpetuity-due has monthly payments where the k-th year are each* $\frac{1}{12}(1+2+\ldots+k)$. *Show that the present value of this perpetuity due is* $\left(\ddot{a}^{(m)}_{\overline{\infty}|i}\right)(I\ddot{a})_{\overline{\infty}|i}$

Theory Assignment 11. *A perpetuity-due has a payment once every k years. The* $j-th$ *payment is* j. *Show that this perpetuity has present value equal to* $\left(\frac{1}{ia_{\overline{k}|i}}\right)^2$.

3 Loans - Chapter 5

Theory Assignment 12. *Prove that the retrospective method and the prospective method for the outstanding balance of an annuity are equivalent. Establish your results at the* k^{th} *payment using an annuity of n level payments of $1 at rate i per period.*

Theory Assignment 13. *Given that a loan has present value of* $a_{\overline{n}|}$, *show that the sum of the interest repaid in the* k^{th} *period plus the principal paid in the* $k+1^{st}$ *period is* $1 + \nu^{n-k}d$.

Theory Assignment 14. *Assume that the present value of a loan is* $a_{\overline{n}|}$ *and show that the sum of the interest paid for any 2 consecutive payments* k^{th} *and* $k = 1^{st}$ *is give by* $2 - (\nu+1)\nu^{n-k}$.

Integrity Expectations

The purpose of this assignment is to help you learn to understand and justify mathematical relationships which will in turn improve your ability to apply those relationships to problems. The actual proofs/arguments already exist and are, therefore, not significant. The learning you do while developing them is critical and valuable.

Therefore using outside sources to complete assignments defeats the purpose and is generally not permitted.

- You may NOT collaborate with other students/people on any of these assignments.
- You may NOT use the internet/other books/other resources to research assignments (or find completed proofs).
- You may NOT use AI on these assignments.
- You may use your textbook, but must make note of this use (reference the page number(s) from the book and very briefly what was helpful)

If you believe there is a valuable source that would help develop your thinking and are not sure if it is permitted, speak with your professor before using it.

By signing below, I attest that I have read, understand, and agree to abide by the integrity expectations outline and implied above.

Signature ______________________________ Date __________

By signing below, I attest that I did abide by the integrity expectations outline and implied above in completing these assignments and that all work presented represents my own intellectual effort and understanding.

Signature ______________________________ Date __________