Ma 391 ~ Operations Research
Multicriteria Homework Problems

I recommend using Excel for all but the algebraic portions of the assignment (# 6 and 7).

Consider

$$\min \left[f_1(x) = e^x, f_2(x) = \begin{cases} \frac{1}{x+1} & 0 \le x \le 5 \\ (x-5)^2 + \frac{1}{6} & x \ge 5 \end{cases} \right]$$
$$\text{subject to } 0 \le x \le 10, x \in \mathbb{R}$$

(this biobjective optimization problem is adapted from problem 2.11 in *Multicriteria Optimization* by Matthias Ehrgott, 2nd Edition, 2005, Springer)

1. Write $f_2(x)$ as a single expression that can be entered into Excel. You will need to use an indicator function, denoted $1_A(x)$.

   $$f_2(x) = \left(\frac{1}{x+1}\right) 1_{x\le 5}(x) + \left((x-5)^2 - \frac{1}{6}\right) 1_{x\ge 5}(x) \text{ where } 1_A(x) = \begin{cases} 0 & x \notin A \\ 1 & x \in A \end{cases}$$

   You may want to consider the use of the `if` function in Excel.

2. Construct the Pareto curve (in the objective space).

3. Graphically identify all the non-dominated points.

4. Based on your graphs, find the set of all efficient solutions.

5. Does this problem have properly efficient solutions? Why or why not?

6. Algebraically state the conditions for a point to be a properly efficient (in the Kuhn Tucker sense) solution.

7. Compute all properly efficient points including the corresponding λ and μ.

8. Formulate the weighted sum scalarization problem. Solve your formulation using the weights,

   $$w_1 = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 \text{ and } w_2 = 1 - w_1$$

   (use Excel's nonlinear solver for each single-objective problem). Graph the Pareto curve of your 10 solutions.

9. Formulate both of the ϵ-constraint method problems for solving the biobjective problem where both f_1 and f_2 are relaxed. Solve your formulations using

   $$\epsilon = (0.01, 0.01), (0.5, 0.1), (1, 0.2), (10, 0.5), (100, 1), (1000, 5), \text{ and } (25000, 30)$$

   (use Excel's nonlinear solver for each single-objective problem - you should have 14 problems to solve). Graph the Pareto curve of your 10 solutions.