Applications of Optimization Paper - 40 points

Due: January 20, 2021

Objective: understand the contexts and types of problems in which optimization is a good tool for the job and those where it is not

Assignment: Write a summary of the strengths and weaknesses of optimization

- characterize the problems for which mathematical optimization works well,
- characterize problems that are actually optimization problems, but the techniques that we have discussed this semester will not work well on them, and
- give some examples from life of each type of problem.

Requirements:

- Briefly research real problems that businesses have. Include appropriate citations using a standard citation method. Include specific page numbers referenced if you reference your textbook.
- minimum length of 1.5 pages (excluding any bibliographic citations)
- standard, 12 point font, single-spaced
- submitted to Canvas assignment by due date

The less trivial the examples, the better your grade will be. The better you demonstrate that you've met the assignment objective, the better your grade will be.

Optimization with Spreadsheets - 20 points

Due: January 25, 2021

Objective: develop the skill to efficiently and professionally set up an optimization problem in Excel with the goal of being able to share/print the page and clearly communicate the problem and the solution; develop skills that will be helpful in solving the case study problems

Assignment: Work the following problems from Chapter 21: 21-1, 21-2, 21-4, 21-5

Upload a pdf for each problem (to represent the "printed" report of your solution) and a single Excel file (with each problem presented/solved on a separate tab) to the Canvas assignment.

You will graded on the accuracy of your answer as well as the professionalism and clarity of your presentation in the Excel file as well as the "printouts."

Multicriteria Optimziation Assignment - 20 points

Due: April 16, 2021

I recommend using Excel for all but the algebraic portions of the assignment (#6 and 7). Present all work professionally (handwritten portions are acceptable).

Consider

$$\min \left[f_1(x) = e^x,\ f_2(x) = \begin{cases} \frac{1}{1+x} & x < 5 \\ (x-5)^2 + \frac{1}{6} & x \geq 5 \end{cases} \right]$$

$$\text{subject to } \ 0 \leq x \leq 10,\ x \in \mathbb{R}$$

(this biobjective optimization problem is adapted from problem 2.11 in *Multicriteria Optimization* by Matthias Ehrgott, 2^{nd} Edition, 2005, Springer)

1. Write $f_2(x)$ as a single expression that can be entered into Excel. You will need to use an indicator function, denoted $1_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{else} \end{cases}$.

   $$f_2(x) = \left(\frac{1}{1+x}\right) 1_{x<5}(x) + \left((x-5)^2 + \frac{1}{6}\right) 1_{x \geq 5}$$

   You may want to consider the use of the `if` function in Excel.

2. Construct the Pareto curve (in the objective space).

3. Graphically identify all the non-dominated points.

4. Based on your graphs, find he set of all efficient solutions.

5. Does this problem have properly efficient solutions? Why or why not?

6. Algebraically state the conditions for a point to be a properly efficient (in the Kuhn Tucker sense) solution.

7. Compute all properly efficient points including the corresponding λ and μ.

8. Formulate the weighted sum scalarization problem. Solve your formulation using the weights,

   $$w_1 = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 03.8, 0.9, 1 \quad \text{and} \quad w_2 = 1 - w_1$$

   (use Excel's nonlinear solver for each single-objective problem). Graph the Pareto curve of your 10 solutions.

9. Formulate both of the ϵ-constraint method problems for solving the biobjective problem where both f_1 and f_2 are relaxed. Solve your formulations using

   $$\epsilon = (0.01, 0.01), (0.5, 0.1), (1, 0.2), (10, 0.5), (100, 1), (1000, 5), and (25000, 30)$$

   (use Excel's nonlinear solver for each single-objective problem - you should have 14 problems to solve). Graph the Pareto curve of your 10 solutions.

Summary of Optimization Techniques - 20 points

Due: April 30, 2021

For each of the algorithms we discussed this semester, briefly characterize the type of problem that is solved using that technique.

- Your characterization should enable you to correctly classify optimization problems.
- You may need to include references to types of variables, constraints on variables, types of constraints (both mathematically and contextually), objective functions, etc.
- You should include strengths and weaknesses of the algorithm.

No minimum length – I would recommend some type of chart format. Complete sentences are not necessary, but ideas should still be clearly communicated. Upload a clear/neat scan of your summary to Canvas by end of day.