Hypothesis Testing

Means, Proportions, Variances for 1 & 2 Populations Examples

TTDD(N)PTT = thus the data do (not) prove that the

- Given the following data: $n_X = 5, \bar{x} = 3.6, s_X = 0.8$ and $n_Y = 11, \bar{y} = 4.2, s_Y = 1.5$. Are these averages truly different at 0.05 level of significance?

  Assumptions: RS, normal population (since $n_i < 30$), independent

  $$P\left(T \leq \frac{3.6 - 4.2 - (0)}{\sqrt{\frac{4(0.64)+10(2.25)}{14}\left(\frac{1}{5}+\frac{1}{11}\right)}} = -0.831469\right) = 0.209832$$

  Do not reject H_0: TTDDNPTT is a difference between the two means with 95% confidence. (There is no provable evidence that the means differ).

  Can we assume homoscedacity ($\sigma_X = \sigma_Y$)? $\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{S_1^2}{S_2^2} \sim F(4, 10)$

  $$P\left(\frac{S_1^2}{S_2^2} \text{ moving away from } 1\right) = P\left(\frac{S_1^2}{S_2^2} \leq \frac{0.8^2}{1.5^2} = 0.284444\right) = 0.118$$
  $$invF(0.025, 4, 10) = 0.113$$

  Do not reject H_0: TTDDNPTT pop var differ with 95% confidence. (There is evidence to justify homoscedacity).

- #13.44 In a study of the effectiveness of certain exercises in weight reduction, a group of 16 persons engaged in these exercises for one month and showed the following results:

  Weight before: 211, 180, 171, 214, 182, 194, 160, 182, 172, 155, 185, 167, 203, 181, 245, 146

  Weight after: 198, 173, 172, 209, 179, 192, 161, 182, 166, 154, 181, 164, 201, 175, 233, 142

  Use the 0.05 level of significance to test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$. Are the exercises effective in weight reduction?

  Samples are not independent, therefore don't use a two population test, consider d =before-after; $H_0 : \mu_d = 0$, $H_1 : \mu_d > 0$

  Compute data: $\bar{x} = 4.125, s_X = 4.06407$

  Assumptions: RS (of the 16 people), normal population

  $$P\left(T \geq \frac{4.125 - 0}{\sqrt{4.06407^2/16}}\right) = 0.000513 < \alpha = 0.05$$
  $$\text{test statistic } t = 4.05997 > 1.75305 = invt(0.95, 15)$$

  Reject H_0: TTDDPTT exercise program decreases weight with 95% confidence.

- #13.72 In a random sample of 200 persons who skipped breakfast, 82 reported that they experienced midmorning fatigue, and in a random sample of 300 people who ate breakfast, 87 reported that they experienced midmorning fatigue. Use the 0.05 level of significance to test $H_0 : p_b = p_{nb}$ against the alternative hypothesis that midmorning fatigue is more prevalent among persons who skip breakfast ($H_1 : p_{nb} > p_b$).

  Assumptions: RS, $n_b p_b, n_b(1 - p_b), n_{nb} p_{nb}, n_{nb}(1 - p_{nb}) > 5$
  since we don't know p_b, p_{nb} use average $p = \frac{169}{500} = 0.338$, so $500(0.338) = 169, 500(0.662) = 331 > 5$

  $$P\left(Z \geq \frac{0.41 - 0.29 - 0}{\sqrt{0.338(0.662)(1/200 + 1/300)}}\right) = 0.002727 < \alpha = 0.05$$
  $$\text{test statistic } z = 2.77897 > 1.64485 = invN(0.95, 0, 1)$$

  Reject H_0: TTDDPTT breakfast decreases midmorning fatigue with 95% confidence.